Probable Entropic Nature of Gravity in Ultraviolet and Infrared Limits. I.Ultraviolet Case

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Abstract

This work presents a study of the possibility for extending the well-known results of E.Verlinde concerning the entropic nature of gravity to the ultraviolet region (Planck's energies) and also the derivation of quantum corrections to Einstein Equations.

1 Introduction

In the last 15-20 years new very interesting approaches to gravity studies have been proposed, which may be divided into "thermodynamical" and "theoretical-informational" approaches. The approach suggested in the pioneer work by T. Jacobson [1] has been considerably extended in a series of remarkable papers by T.Padmanabhan [2]–[12]. The paper by E.Verlinde [13]stating a secondary character of the gravitational interaction and its entropic nature was published in 2010 after the appearance in the ArXiv. The paper [13] introducing such specific terms as "Entropic Force" has been followed by numerous studies (e.g. [14] and others). Note, however, that the notion of Entropic Force, without the introduction of the term per se, has been proposed in Conclusion of [15] earlier than in [13].
In this work the author studies the possibility for extension of the results given in [13] to the ultraviolet region (Planck's energies) and presents the

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derivation of quantum corrections to Einstein Equations using the dimensionless small parameter α introduced by the author in his previous works [16] – [28].

2 Fundamental Quantities and Their High-Energy Deformation

In this section the author uses the "ideology", terms, and notation introduced in [13] to extend the corresponding results to the ultraviolet and infrared gravity regions. It may be stated that the results in [13] have been obtained for the "medium" energies, i.e. for the range of well-known energies, where the General Relativity (GR) is valid. But owing to modern knowledge, in the ultraviolet and infrared limits gravity may be modified. As regards the ultraviolet (Planck) scale, this idea has been proposed long ago [29] –[34] and in this situation the word "may" we have to replace by the word "must". As for low energies, there are many recent publications considering the infrared (for great distances) modification of gravity (e.g.[35]). And the modification can have a solid experimental status in the nearest future [36].
Naturally, when we are concerned with extension of some results to higher or lower energies, the principle of conformity must be executed undeviatingly: on going to the known energy scales the known results must be reproduced.
In this Section it is shown that the fundamental quantities A,N,T, defined in [13] and associated with the holographic screen (where A - its surface area, N - number of data bits "existing" on A, T – its temperature), may be supplementary defined for the region of high energies so that on going to normal energies they be coincident with the values given in [13]. In the process we take the corresponding quantities for the stationary Schwarzshild black hole as most natural holographic object.
The idea is as follows: formulae for the correction of the fundamental quantities, excepting the temperature T, within the Generalized Uncertainty Principle (GUP) for the holographic screen $\mathcal{S}$ from [13] are similar to those for the black hole. For the classical temperature in [13] an original formula has been found ((5.28) from [13]), the GUP-corrections of which may be de-

rived in analogy with the GUP-corrections presenting quantum corrections to the corresponding semiclassical formula for temperature of a large black hole with the radius $R \gg l_p$.

Let us consider in [13] Section 5.2 (Derivation of Einstein Equations). In this Section formula (5.32) for a "bit density" on the holographic screen is given as

$$dN = \frac{dA}{G\hbar}. \tag{1}$$

However, when the holographic principle [37]–[40] is valid, N is actually the entropy S up to the factor $S \sim N$ and hence from (1) it follows directly that

$$dS \sim \frac{dA}{G\hbar}. \tag{2}$$

What are the changes in S on going to high (Planck) energies? The answer to this question is already known owing to the fact that at these energies the Heisenberg Uncertainty Principle (HUP) is replaced by GUP [41]–[45]:

$$\triangle x \geq \frac{\hbar}{\triangle p} + \ell^2 \frac{\triangle p}{\hbar}, \tag{3}$$

where $\ell^2 = \alpha' l_p^2$ and α' – dimensionless numerical factor. The well-known Bekenstein-Hawking formula for the black hole entropy in the semiclassical approximation [46],[47]

$$S^{BH} = \frac{A}{4l_p^2} \tag{4}$$

is modified by the corresponding quantum corrections on going from HUP to GUP [48]–[51] as follows:

$$S_{GUP}^{BH} = \frac{A}{4l_p^2} - \frac{\pi\alpha'^2}{4} \ln\left(\frac{A}{4l_p^2}\right) + \sum_{n=1}^{\infty} c_n \left(\frac{A}{4l_p^2}\right)^{-n} + \mathrm{const}\ , \tag{5}$$

where the expansion coefficients $c_n \propto \alpha'^{2(n+1)}$ can always be computed to any desired order of accuracy.

In this way at high energies we have $S \rightarrow S_{GUP}$ and hence $N \rightarrow N_{GUP}$. Assuming in the notation of [13] that

$$S = \frac{1}{4}N, \tag{6}$$

we directly obtain

$$N_{GUP} = \frac{A}{l_p^2} - \pi\alpha'^2 \ln\left(\frac{A}{4l_p^2}\right) + 4\sum_{n=1}^{\infty} c_n \left(\frac{A}{4l_p^2}\right)^{-n} + \text{const} \ . \tag{7}$$

Now, coming back to [13], in terms of N_{GUP} we can define the holographic screen area, as measured at high energies, by

$$A_{GUP} \equiv G\hbar N_{GUP}, \tag{8}$$

where G and $\hbar$ – gravitational and Planck constants, respectively, and N_{GUP} is given (7). Considering that we, similar to [13], assume that the speed of light $c = 1$, then, according to $l_p^2 = G\hbar$, from (7) and (8) we have

$$A_{GUP} = A - G\hbar\pi\alpha'^2 \ln\left(\frac{A}{4G\hbar}\right) + 4G\hbar\sum_{n=1}^{\infty} c_n \left(\frac{A}{4G\hbar}\right)^{-n} + \text{const} \ . \tag{9}$$

In this case an exact value of the constant in the right-hand side of (9) is of no great importance as further we need the relation (1), being primarily interested in dA_{GUP} rather than in A_{GUP}, i.e. the constant in the right-hand side of (9) is insignificant. So, (1) has a fairly definite analog at high energies

$$dN_{GUP} = \frac{dA_{GUP}}{G\hbar} \tag{10}$$

that on going to the known low energies gives (1). There is a single considerable difference, in [13] the quantity N was defined in terms of A and dN was defined in terms of dA but in the case under study the situation is opposite: A_{GUP} is defined in terms of N_{GUP} and dA_{GUP} in terms of dN_{GUP}. The logic series is here as follows:

$$A \Rightarrow N \Rightarrow N_{GUP} \Rightarrow A_{GUP}. \tag{11}$$

Now we proceed to high-energy redefinition of the temperature T for the holographic screen. As a basis, we again take the semiclassical temperature T^{BH} of a black hole and its high-energy generalization T^{BH}_{GUP} within the scope of GUP. As is known,

$$T^{BH} = \frac{\hbar c^3}{8\pi kGM} = \frac{\hbar}{8\pi GM}, \tag{12}$$

where, among the earlier undefined quantities, k — Boltzmann constant that, similar to the speed of light c, is set to 1; M — mass of a black hole. But we need the expression for T^{BH} and T^{BH}_{GUP} of a somewhat other (equivalent)form as derived in ([48], formulae (10) and (11)).
Using the notation and results from [48] and assuming that $d = 4$ is a space-time dimension, r_+ is a radius of the event horizon, in this case given by

$$r_+ = [16\pi GM/(d-2)\Omega_{d-2}]^{1/(d-3)} = [16\pi GM/2\Omega_2] = 2\pi GM, \qquad (13)$$

where Ω_{d-2} is the area of the unit sphere S^{d-2}. From this it follows directly that (12) may be rewritten as

$$T^{BH} = \frac{\hbar}{8\pi GM} = \frac{\hbar}{4\pi r_+}. \qquad (14)$$

On going from T^{BH} to T^{BH}_{GUP}, the contribution of T^{BH} may be derived explicitly. Specifically, for large black holes with $r_+ \gg \alpha' l_p$ we have ([48], formula (11)), where the points in the right-hand side (15) denote the terms of a higher order of smallness

$$\begin{aligned} T^{BH}_{GUP} &= \frac{1}{4\pi}[\frac{\hbar}{r_+} + \frac{\hbar\alpha'^2 l_p^2}{r_+^3} + \frac{\hbar\alpha'^4 l_p^4}{r_+^5} + ...] \\ &= T^{BH} + \frac{\hbar}{4\pi r_+}[\frac{\alpha'^2 l_p^2}{r_+^2} + \frac{\alpha'^4 l_p^4}{r_+^4} + ...] \\ &= T^{BH} + \widetilde{T}^{BH}_{GUP}. \end{aligned} \qquad (15)$$

It is obvious that on going to the semiclassical approximation this formula results in T^{BH}. The terms of a series in the right-hand side (15) may be calculated in any order. As we start with a completely classical treatment, in a similar manner we first consider the case of a large screen $\mathcal{S}$ [13] with a characteristic size (radius)

$$R \gg \alpha' l_p. \qquad (16)$$

Without any loss in the generality, we can assume that the screen $\mathcal{S}$ is

spherically symmetric and with the radius R. Then we can redefine T for high energies from formulae ((5.28), (5.33) [13]) in analogy with (15) using GUP as follows:

$$T_{GUP} = T + \frac{\hbar}{4\pi R}[\frac{\alpha'^2 l_p^2}{R^2} + \frac{\alpha'^4 l_p^4}{R^4} + ...] = T + \widetilde{T}_{GUP}. \qquad (17)$$

Thus, we can have a GUP - analog of Komar's mass in ((5.33) from [13])

$$M_{GUP} = \frac{1}{2}\int_{\mathcal{S}} T_{GUP} dN_{GUP} = \frac{1}{2G\hbar}\int_{\mathcal{S}} T_{GUP} dA_{GUP}, \qquad (18)$$

that in the low-energy limit gives the well-known Komar formula [52], ([53], p.289).

Note that in this case, like in formula (5.34) from [13], the Plank constant $\hbar$ before the basic formula disappears finally. Indeed,

$$\begin{aligned} M_{GUP} &= \frac{1}{2G\hbar}\int_{\mathcal{S}} T_{GUP} dA_{GUP} \\ = \frac{1}{2G\hbar}\int_{\mathcal{S}}(T + \frac{\hbar}{4\pi R}[\frac{\alpha'^2 l_p^2}{R^2} &+ \frac{\alpha'^4 l_p^4}{R^4} + ...]) dA_{GUP} \\ &= \frac{1}{4\pi G}\int_{\mathcal{S}} e^{\phi}\nabla\phi dA_{GUP} \\ +\frac{1}{8\pi RG}\int_{\mathcal{S}}[\frac{\alpha'^2 l_p^2}{R^2} &+ \frac{\alpha'^4 l_p^4}{R^4} + ...] dA_{GUP}, \end{aligned} \qquad (19)$$

where by (5.28) of [13] $T = \frac{\hbar}{2\pi} e^{\phi} N^b \nabla_b \phi$ – "classical" temperature $\mathcal{S}$. However, $\hbar$ by (9) is preserved in the measure of dA_{GUP} and completely in the second part of the last formula as in the case studied we have $l_p^2 = G\hbar$.

It is clear that the "GUP-deformed Komar's mass" M_{GUP} in the first term (19) as a summand has the known Komar's mass [52],((11.2.9 - 11.2.10),[53]) ((5.34), [13])

$$M = \frac{1}{4\pi G}\int_{\mathcal{S}} T dA. \qquad (20)$$

3 N_{GUP},A_{GUP}, T_{GUP} , and M_{GUP} in Terms of Unified Small Parameter

If feasible, it is desirable to express all the above-derived fundamental quantities in terms of a unified parameter. As shown by the author in [28], [54], this is possible for black holes within the scope of GUP and a role of the unified small parameter is played by the parameter introduced previously in [16]–[25] as follows:

$$\alpha_x = l_{min}^2/x^2, \tag{21}$$

where x is the measuring scale, $l_{min} \sim l_p$ by virtue of GUP (3), and $0 < \alpha \leq 1/4$.
Obviously, the principal results obtained in [28], [54] remain in force for an arbitrary screen $\mathcal{S}$ and may be applied to the quantities N_{GUP},A_{GUP}, T_{GUP} defined in the preceding Section.
N_{GUP}, in particular, is of the form

$$N_{GUP} = N - \pi\alpha'^2 \ln(\sigma\alpha_R^{-1}) + 4\sum_{n=1}^{\infty}(c_n\sigma^{-n})\alpha_R^n + \text{const}, \tag{22}$$

where R – characteristic linear size (radius) of the screen $\mathcal{S}$; α_R – value of α parameter at the point R, σ is a dimensionless computational factor.
It is convenient to refer to the form N_{GUP} derived in (22) α as to representation. Using (8) and (9), we can easily obtain α – representation for A_{GUP}

$$A_{GUP} = A - \pi G\hbar\alpha'^2 \ln(\sigma\alpha_R^{-1}) + 4G\hbar\sum_{n=1}^{\infty}(c_n\sigma^{-n})\alpha_R^n + \text{const}. \tag{23}$$

In a similar way, we can write T_{GUP} (17) as

$$T_{GUP} = T + T[\alpha_R], \tag{24}$$

where $T[\alpha_R]$ – part dependent only on α_R and fundamental constants G and $\hbar$, so we have

$$\lim_{\alpha_R \to 0} T[\alpha_R] = 0. \tag{25}$$

Also, it is clear that M_{GUP} (18),(19) may be derived in terms of α_R.
Note that the appearance of α_x in (22),(23),(24) is not accidental. Here α_x is considered as a deformation parameter for the Heisenberg algebra on going from HUP to GUP. Generally speaking, initially the construction of such a deformation was realized with other parameters (e.g. [44],[45]). But it is easily shown that QFT parameter of the deformations associated with GUP may be expressed in terms of the parameter α that has been introduced in the approach to the density matrix deformation [27],[28]. Here the notation of [55] is used. Then ([28], p. 943)

$$[\vec{x},\vec{p}] = i\hbar(1+\beta^2\vec{p}^2+...) \tag{26}$$

and

$$\Delta x_{\min} \approx \hbar\sqrt{\beta} \sim l_p. \tag{27}$$

In this case from (26),(27) it follows that $\beta \sim \mathbf{1/p^2}$, and for $x_{\min} \sim l_p$, β corresponding to $x_{\min}$ is nothing else but

$$\beta \sim 1/P_{pl}^2, \tag{28}$$

where P_{pl} is Planck's momentum: $P_{pl} = \hbar/l_p$.
In this way β is varying over the following interval:

$$\lambda/P_{pl}^2 \leq \beta, \tag{29}$$

where λ is a numerical factor and the second term in (26) is accurately reproduced in the momentum representation (up to the numerical factor) by $\alpha_x = l_{min}^2/x^2 \sim l_p^2/x^2 = p^2/P_{pl}^2$

$$[\vec{x},\vec{p}] = i\hbar(1+\beta^2\vec{p}^2+...) = i\hbar(1+a_1\alpha_x+a_2\alpha_x^2+...). \tag{30}$$

In the case under study convenience of using α_x stems from its smallness, its dimensionless character, and ability to test changes in the radius R of the holographic screen $\mathcal{S}$.

4 Quantum Corrections to the Principal Result and Ultraviolet Limit

Based on the aforesaid, we can proceed to generalization of the results from Section 5.2 of [13] and to derivation of equations for a gravitational field

within the scope of GUP.
We must consider two absolutely different cases.

4.1 Quantum Corrections to the Principal Result

It is assumed that the screen radius is given by $\mathcal{S}$

$$R \gg l_p, \tag{31}$$

that is actually equivalent to (16), because α' is a numerical factor on the order of 1.
In terms of the deformation parameter α_x introduced in the previous Section, we have

$$\alpha_R \ll 1/4. \tag{32}$$

So far we are not concerned with redefinition of the lower limit for α_R. This is interesting when going to the infrared limit.
Then the principal result from the final part of Section 5.2 in [13] remains valid owing to the replacement of M (formula (5.34) from [13]) by $M_{GUP} = M_{GUP}[\alpha_R]$ (18),(19). The "α_R – complement" (i.e. the difference $\widetilde{M}[\alpha_R] = M_{GUP} - M$) to M will be simply a (small) quantum correction for the principal result.
Because of (31) and (32), it is supposed that α_R is continuously varying from R and all the quantities in Section 5.2 of [13] are also continuously dependent on α_R (32). Then we can write down the ("α – analog" of formula (5.37) in [13])as

$$2\int_\Sigma \left(T_{ab}[\alpha] - \frac{1}{2}T[\alpha]g_{ab}[\alpha]\right) n^a \xi^b dV = \frac{1}{4\pi G}\int_\Sigma R_{ab}[\alpha] n^a \xi^b dV, \tag{33}$$

where the dependence of $T_{ab}[\alpha]$ and $R_{ab}[\alpha]$ on $\alpha = \alpha_R$ is completely determined, in accordance with [53],[13], by the integral $M_{GUP}[\alpha]$ (19).
Besides, it is assumed that n^a and ξ^b are dependent on α, though the dependence is dropped.
Next, similar to [13], from (33) we can derive the **α-deformed** Einstein Equations using the method from [1]. Note that both this method and its

minor modification given in ([13], end of Section 5.2) in this case are valid because α_R is small and continuous, the whole system being continuously dependent on it.
Solutions of the α**-deformed** Einstein Equations represent a series in α_R, and for $\alpha_R \to 0$ or for $\alpha' = 0$ become the corresponding solutions of (Section 5.2 in [13]).
Using the result obtained in [56], we can easily extend the above result to the case with a nonzero cosmological term $\Lambda \neq 0$. In [56] Komar's formula was generalized to the case of a nonzero Λ. All the arguments from (Section 5.2 of [13]) in this case remain valid and formula (5.37) takes the following form:

$$2\int_{\Sigma}\left(T_{ab}-\frac{1}{2}Tg_{ab}\right)n^a\xi^b dV = \frac{1}{4\pi G}\int_{\Sigma}(R_{ab}+\Lambda g_{ab})n^a\xi^b dV. \qquad (34)$$

We can easily obtain the α - analog of the last formula with the dynamic cosmological term $\Lambda(\alpha)$ as a corresponding complement to the right-hand side (33). Analysis of the relationship between Λ and α, applicable in this case as well, will be given in Subsection 4.2.

4.2 Ultraviolet Limit

In the case in question we suggest that the screen $\mathcal{S}$ has a radius on the order of several Planck's lengths

$$R \approx \xi l_{min} = 2\alpha' \xi l_p, \qquad (35)$$

where ξ – number on the order of 1 or

$$\alpha_R \approx 1/4. \qquad (36)$$

The problem is which object puts the limit for such a screen $\mathcal{S}$. It may be assumed that if $T_{ab} \neq 0$ then the object may be represented only by Planck's black hole or by a micro-black hole with a radius on the order of several Planck's lengths.
Clearly, the methods of [13] and [1] are not in force for such screen $\mathcal{S}$.

Specifically, there are no classical analogs of N, T , and M for the screen. Moreover, it is impossible to use the result of [1] as "a very small region the space-time" is no longer "an approximate Minkowski space" [13].
Also, such micro-black hole is a horizon space, jet at high energies (Planck scales). As is known, for horizon spaces, black holes in particular, at low energies (semiclassical approximation) the results of [12] are valid.
At the horizon (and we are interested in this case only) Einstein's field Equations may be written as a thermodynamic identity ([12] formula (119))

$$\underbrace{\frac{\hbar c f'(a)}{4\pi}}_{k_B T} \underbrace{\frac{c^3}{G\hbar} d\left(\frac{1}{4} 4\pi a^2\right)}_{dS} \underbrace{-\frac{1}{2}\frac{c^4 da}{G}}_{-dE} = \underbrace{P d\left(\frac{4\pi}{3} a^3\right)}_{P\,dV} \tag{37}$$

where $R = a$ – radius of a black hole(i.e. of the screen $\mathcal{S}$)), $P = T_R^R$ is the trace of the momentum-energy tensor and radial pressure, and the horizon location will be given by simple zero of the function $f(R)$, at $R = a$.
The main ingredients of (37) may be written in terms of the deformation parameter α with the coefficients containing only the numerical factors and fundamental constants [54].
Also, the work [54] presents two possible variants of high-energy (Planck) α -deformation $\alpha \to 1/4$ (37).
Hereinafter, we assume that the energy – momentum tensor of matter fields is not traceless

$$T_a^a \neq 0, \tag{38}$$

similar, in particular, to the case under study (37) $P = T_R^R \neq 0$

1. Case of equilibrium thermodynamics ([54], section (6.1))

In this case it is assumed that in the high-energy (ultraviolet (UV))limit the thermodynamic identity (37) is retained but now all the quantities involved in this identity become α-deformed ($\alpha \to 1/4$). All the quantities Υ in (37) are replaced by the corresponding quantities Υ_{GUP} with the subscript GUP. Then the high-energy α-deformation of equation (37) takes the form

$$k_B T_{GUP}(\alpha) dS_{GUP}(\alpha) - dE_{GUP}(\alpha) = P(\alpha) dV_{GUP}(\alpha). \tag{39}$$

Substituting into (39) the corresponding quantities $T_{GUP}(\alpha), S_{GUP}(\alpha), E_{GUP}(\alpha), V_{GUP}(\alpha), P(\alpha)$ and expanding them into a Laurent series in terms of α, close to high values of α, specifically close to $\alpha = 1/4$, we can derive a solution for the high energy α-deformation of the general relativity (39) as a function of $P(\alpha)$. Provided at high energies the generalization of (37) to (39)is possible, we can have the high-energy α-deformation of the metric.
It is noteworthy that in (39) T_{GUP} this time is calculated from ([48], formula (10))

$$T_{GUP}^{BH} = \frac{1}{4\pi}\frac{\hbar R}{2\alpha'^2 l_p^2}[1 - \sqrt{1 - \frac{\alpha'^2 l_p^2}{R^2}}] = \frac{\hbar \alpha_R^{-1}}{4\pi\alpha' l_p}[1 - (1-\alpha_R)^{1/2}] \tag{40}$$

(with subsequent replacement of l_p by $\sqrt{G\hbar}$) for $c = 1$) as formula (15) in the case of micro-black holes is no longer valid.
It is especially interesting to consider the following case.

1. Case of nonequilibrium thermodynamics ([54], section (6.2))

In this case the α - dependent dynamic cosmological term $\Lambda(\alpha) \neq 0$ appears in the right-hand side of(39). Then, with the addition of $\Lambda(\alpha) \neq 0$, the α − representation (39)(for $\hbar = 1$) is given as follows(([54], formula (53)):

$$-\alpha^2 f'(\alpha) - \frac{1}{2}\alpha = 16\pi\alpha'^2 P(\alpha) G^2 - G\Lambda(\alpha), \tag{41}$$

where $\alpha = \alpha_R \approx 1/4$,

$$f'(\alpha) = 4\pi k_B T_{GUP}(\alpha) \tag{42}$$

and the derivative in the left-hand side of (42) is taken with respect to α.
$\Lambda(\alpha)$ in the right-hand side of (42) may be subjected to a series expansion in terms of α, in compliance with the holographic principle [37]–[40] as applied to the Universe [57]. In [58],[27], [28],[54] in the leading order this expansion results in the first power, i.e. we have

$$\Lambda(\alpha_R) \sim \alpha_R \Lambda_p, \tag{43}$$

where Λ_p – initial value of $\Lambda \approx \Lambda_{1/4}$ derived using the well-known procedure of "summation over all zero modes " and the Planck momentum cutoff [59],[60]. Actually, (43)is in a good agreement with the observable $\Lambda = \Lambda_{observ}$. Because a radius of the visible part of the Universe is given as $R = R_{Univ} \approx 10^{28} cm$, it is clear that $\alpha_R \approx 10^{-122}$ and (43)is completely consistent with the experiment [60].

Note that, proceeding directly from a quantum field theory but without the use of the holographic principle, we can have only a rough estimate of Λ that, on the whole, is at variance with Λ_{observ}. Such an estimate may be obtained in different ways: by simulation [61]; using the cutoff [59] but now in the infrared limit; with the use of the Generalized Uncertainty Principle for the pair (Λ, V), where V – four-dimensional volume [27], [28]. In the α–representation in this case the expansion in terms of α results in the second leading order

$$\Lambda(\alpha_R) \sim \alpha_R^2 \Lambda_p, \tag{44}$$

that, obviously, is at variance with the accepted facts.

5 Conclusion

I. A very interesting case of the zero energy-momentum tensor for matter fields $T_{ab} = 0$ (and , specifically the case of $P(\alpha) = 0$) in the right-hand side of (41) has remained beyond the scope of the final Section. We can state the problem more specifically: for which conditions in this case we can derive a solution in the form of the de Sitter space with large values of α? This problem is also important when we try to find whether it is possible to derive the initial inflation conditions [62],[63] for $T_{ab} = 0$ on the basis of the foregoing analysis.

Note that the dynamic cosmological term $\Lambda(\alpha)$ correlates well with inflation models [62],[63] as the latter require a very high Λ at the early stages of the Universe, and this is distinct from $\Lambda = \Lambda_{exper}$ in the modern period. Of great interest is the recent work [64], where a mechanism of the vacuum energy decay in the de Sitter space is established to support a dynamic

nature of Λ.
II. The deformation parameter α_R has a double meaning.
As $\kappa = 1/R$ – curvature with the radius R, $\alpha_R = \kappa^2 l_{min}^2$ is nothing else but the squared curvature multiplied by the squared minimal area and is explicitly dependent on the energy E. On the other hand, it is seen that, at least at the known energies, from the definition of the bit number N in ([13], formula (3.10)) we get $\alpha_R \sim 1/N$. But, because $\alpha_R = \alpha_R[E]$, this suggests that $N = N[E]$, as demonstrated in the text on going to higher energies

$$N \Rightarrow N_{GUP}. \tag{45}$$

Nevertheless, on going to lower energies, i.e. in the infrared limit, the same should be true: the bit number must be a function of energy.
This problem and the relevant questions touched upon in this paper will be further considered in subsequent works of the author.

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